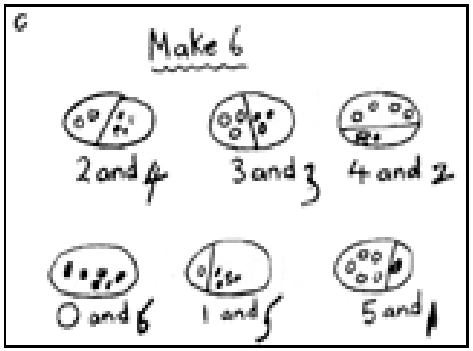
Progression in Written Addition

Stage 1



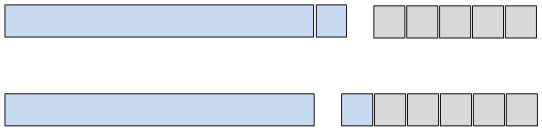
Children are taught that addition is the combining of two or more amounts. They begin by counting all of the items in the groups, then move on to counting on from the largest amount. Children are encouraged to develop a mental image of the size of numbers. They learn to think about addition as combining amounts in practical, real life situations.
They begin to record addition number sentences such as
2 + 4 = 6 and 8 = 3 + 5 and 3 + 2 + 4 = 9

Stage 2

Children move on to using Base 10 equipment to support their developing understanding of addition.

11 + 5 = 16

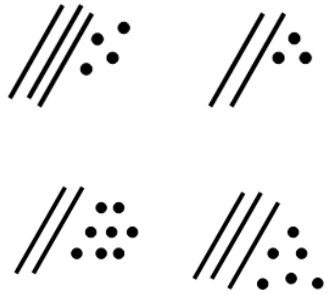
11 cubes are lined up (1 ten and 1 one).
5 cubes are added to the line of 11 giving a total of 16.



If possible, use two different colours of base 10 equipment so that the initial amounts can still be seen.

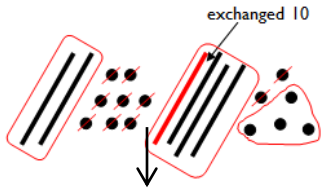
Stage 3

Children continue to use the Base 10 equipment or place value counters to support calculations, exchanging 10 ones for 1 ten when the total of the ones is 10 or more. They will record their own drawings of the Base 10 equipment, using lines for 10 rods and dots for the ones blocks.



34 + 23 = ?
The ones are added first 4 + 3 = 7
The tens are added next 30 + 20 = 50
Both answers are put together 50 + 7 = 57

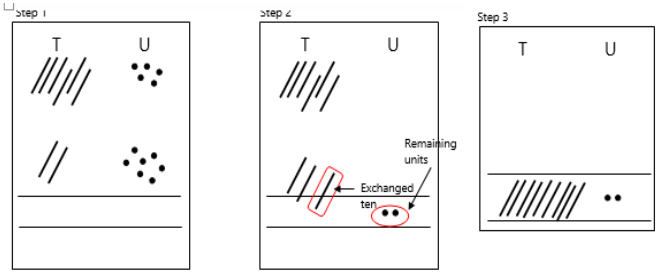
28 + 36 = ?
The ones are added first 8 + 6 = 14 with ten ones exchanged for 1 ten.
A ring is put around the ones not exchanged –



this is the ones part of the answer. The tens are then added, including the exchanged ten, to complete the sum.

Stage 4

65 + 27



Written method

Step 1	Step 2	Step 3
T U 6 5 + 2 7	T U 6 5 + 2, 7 2	T U 6 5 + 2, 7 9 2
Write out calculation.	Add ones	Add tens

Stage 5

$$\begin{array}{r} 3\,3\,6\,4 \\ +\quad 2,4,7 \\ \hline 3\,6\,1\,1 \end{array} \qquad \begin{array}{r} 3\,1\,2\,1 \\ 3\,7 \\ +\quad 1,4,8 \\ \hline 3\,3\,0\,6 \end{array} \qquad \begin{array}{r} 3\,.\,5\,6 \\ +\ 2\,.\,4,7 \\ \hline 6\,.\,0\,3 \end{array}$$

$$\begin{array}{r} 4\,0\,1\,.\,2\,0 \\ 2\,6\,.\,8\,5 \\ +\quad 0,.\,7\,1 \\ \hline 4\,2\,8\,.\,7\,6 \end{array}$$

When adding decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2

This is the final stage of the method, and should be continued to be used for all written addition calculations.
Children will be expected to use this method for adding numbers with more than 3 digits, numbers involving decimals and adding any number of amounts together.

Children should not be made to go onto the next stage if:

1) **they are not ready.**
2) **they are not confident.**

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

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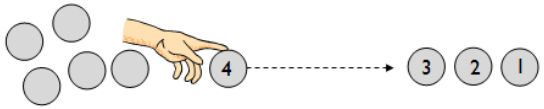
Progression in Written Subtraction

Stage 1

Children will subtract two numbers by taking one away from the other and counting how many are left.



Children are encouraged to develop a mental image of the size of numbers. They learn to think about subtraction as 'take away' in practical, real life situations.
They begin to record subtraction number sentences such as 8 – 5 = 3

Stage 2

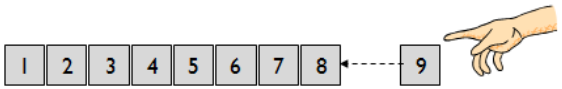
Children move on to using Base 10 equipment alongside a number track to support their developing understanding of subtraction.

13 - 4 = ?

13 cubes are lined up.
4 cubes are removed from the end of the line leaving 9 left.
It is important that children keep track of how many have been removed.

Touch count and remove the number to be taken away.

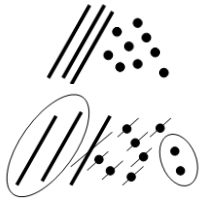
Touch count to find the number that remains.



Stage 3

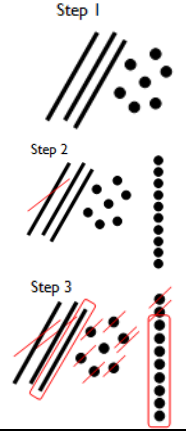
Children continue to use the Base 10 equipment to support their calculations. They will record their own drawings of the Base 10 equipment, using lines for 10 rods and dots for the unit blocks.

39 - 17 = ?

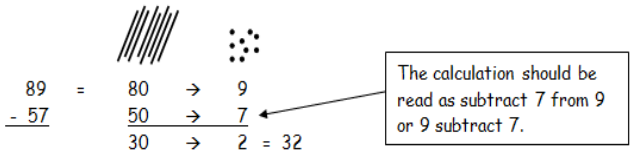


39 is drawn
17 is crossed out
A ring is drawn around what is left to give the answer giving 22

37 – 19 = ?
37 is drawn
9 units cannot be crossed out, so a ten is crossed out and exchanged for 10 ones which are in a line.
19 is crossed out
A ring is drawn around what is left to give the answer 18



Stage 4A



Children move from using the Base 10 method to expanded vertical method, using base 10 notation, place value counters and/or arrow cards.
Children learn to subtract the least significant digits first (start with the numbers on the right and work from right to left).
The answer to each individual subtraction is written underneath before these answers are recombined.

Stage 4B

This stage involves exchange.

It is clear that there are not enough units to subtract 6 from 1, so one of the tens from the 70 is exchanged for 10 ones.

70 → 1
- 40 → 6

The initial number 71 is rearranged as 60 and 11 to make the calculation easier.

60 → 11
- 40 → 6
20 → 5 = 25

This would be recorded by the children as:

60
~~7~~0 → 11
- 40 → 6
20 → 5 = 25

Stage 5

This final stage is the compact method of decomposition. The example shows how the same calculation would be carried out using the previous method and the final method.

754
- 86

Stage 4B

600 140
~~7~~00 → ~~5~~0 → 14
- 80 → 6
600 → 60 → 8 = 668

becomes

Stage 5

6141
~~7~~54
- 86
668

This is the final stage of the process and will continue to be used with larger numbers and numbers involving decimals.

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Progression in Written Multiplication

Stage 1

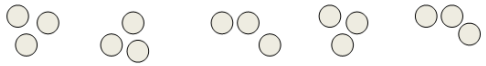
Children are encouraged to develop a mental image of the size of numbers. They learn to think about equal groups or sets of objects in practical, real life situations. They begin to record these situations using pictures.

A child's jotting showing fingers on each hand as a double.

A child's jotting showing double three as three cookies on each plate.

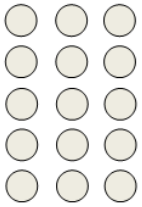
Stage 2

Children understand that multiplication is repeated addition and that can be done by counting in equal steps/groups.



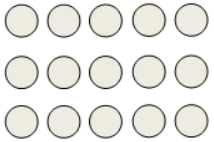
or

Children can then be introduced to the image of a rectangular array, initially through real items such as egg boxes, baking trays, ice cube trays, wrapping paper etc. and using these show that counting up in equal groups can be a quicker way of finding a total.



to

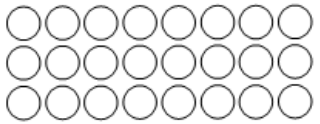
3 + 3 + 3 + 3 + 3 = 15



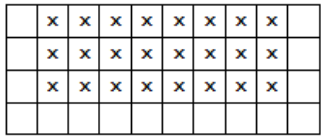
Children also understand that 3 x 5 is the same as 5 x 3

5 + 5 + 5 = 15

Stage 3

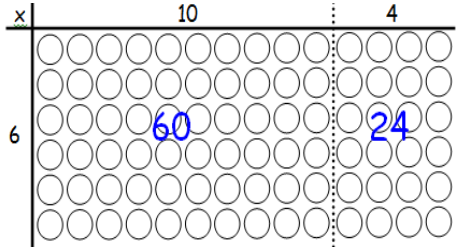


3 x 8 = 8 + 8 + 8 = 24



3 x 8 = 8 + 8 + 8 = 24

Children continue to use arrays and create their own to represent multiplication calculations



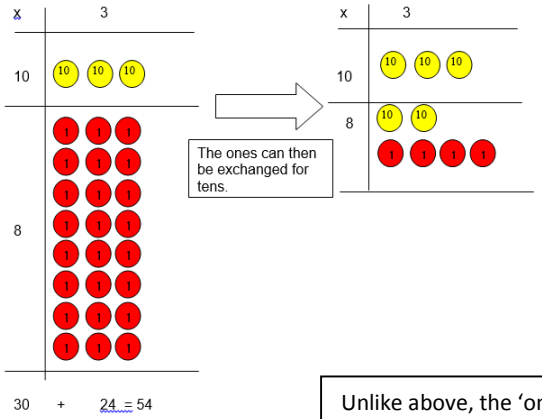
(6 x 10) + (6 x 4)

60 + 24

84

Stage 4

Children will use arrays with place value counters to lead into the written method of multiplication.



This can be written as:

18
X 3
24
30
54

Unlike above, the 'ones' calculation (8x3) is written first, the tens calculation (10x3) underneath.

Stage 5

Children will further develop their knowledge of short multiplication to multiply any two-digit and three-digit by any single-digit number, e.g.

79
X 8
72
560
632

346
X 8
48
320
2400
2768

This is then shortened to:

346
X 38
2 768

For a four-digit by one digit calculation:

4346
X 28
34768

Stage 6

For two digit by two digit children will use the expanded method first using the least significant digit first.

72
x 38
16 (2x8)
560 (70x8)
60 (2x30)
2100 (70x30)
2736

When children are confident this can then be shortened to:

72
x 38
576 (72x8)
2160 (72x30)
2736

For decimal numbers children will start with the expanded method and then progress to the compact method.

4.92
X 2 3
0.06 (0.02 x 3)
2.70 (0.9 x 3)
12.00 (12 x 3)
14.76

4.92
X 2 3
14.76

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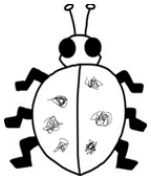
Progression in Written Division

Stage 1

Children are encouraged to develop a mental image of the number system in their heads to use for calculation. They should experience practical calculation opportunities involving **equal** groups and **equal** sharing.

They may develop ways of recording calculations using pictures.

A child's jotting showing halving six spots between two sides of a ladybird.



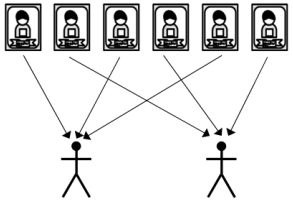
A child's jotting showing how they shared the apples at snack time between two groups.

Stage 2

Children explore practical contexts where they share equally and group equally. 6 ÷ 2 = ?

Equal sharing (6 shared equally between 2)

6 football stickers are shared equally between 2 people, how many do they each get? Children may solve this by using a 'one for you, one for me' strategy until all of the stickers have been given out.



Equal grouping (How many groups of 2 are there in 6?)

There are 6 football stickers, how many people can have 2 stickers each?

Stage 3

Children continue to use practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation.

12 ÷ 3 = ? Children begin to read this calculation as, 'How many groups of 3 are there in 12?'

At this stage, children will also be introduced to division calculations that result in remainders.

13 ÷ 4 = 3 remainder 1

Stage 4

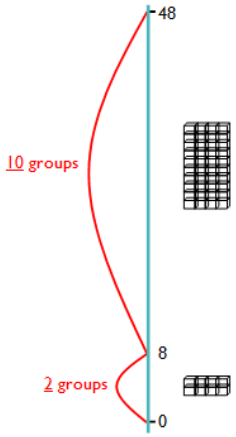
43 ÷ 8

43 ÷ 8 = 5 remainder 3

At this stage, children also learn if the remainder should be rounded up or down e.g. I have 43p. Sweets are 8p each. How many can I buy? Answer: 5 (the remaining 3p is not enough for another sweet)

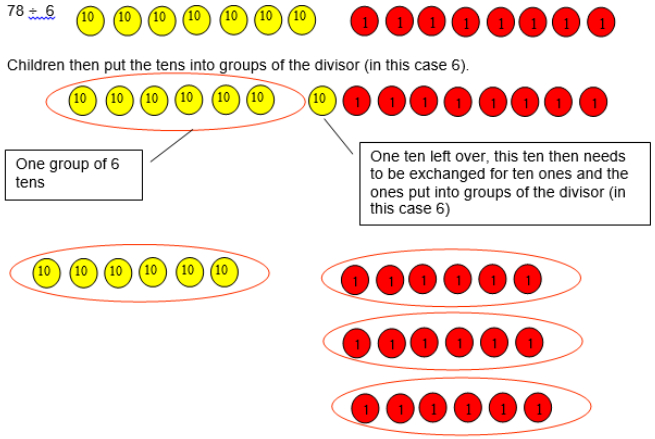
Apples are packed into boxes of 8. There are 43 apples. How many boxes do I need? Answer: 6 (the remaining 3 apples still need a box)

The method of repeated subtraction on a number line is then continued, but using a vertical number line alongside practical equipment.



Stage 5

Children will learn the short method of division using place value counters or Base 10.

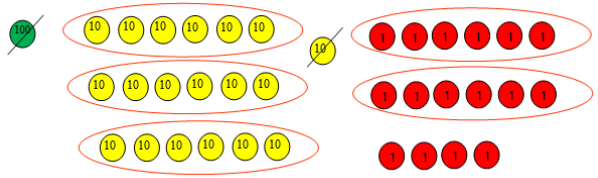


This is then written as

$$6\overline{)7^{1}8}\quad 1\,3$$

Stage 5 continued

196 ÷6



$$6\overline{)1^{1}9^{1}6}\quad 3\ 2r4$$

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Progression in Written Division

Stage 6

Children will move away from using manipulatives when they are confident with the process involved in dividing. At this point, when developing their understanding of short division, children could utilise a 'key facts' box, as shown below. This enables an efficient recall of tables facts.

523 ÷ 8

$$8\overline{)5^{5}2^{4}3} \quad 6\ 5\text{r}3$$

Key facts box 8x

1x	8
2x	16
5x	40
10x	80

Children will then use short division method to divide a four digit number by a single digit number. If children still need to use the key facts box, it can be extended to include 100x.

2458 ÷ 7

$$7\overline{)2^{2}4^{3}58} \quad 3\ 5\ 1\text{r}1$$

Stage 6 continued

Short division with fraction or decimal remainders:

362 ÷ 16

$$16\overline{)3^{3}6^{4}2.^{10}0^{4}0} \quad 2\ 2.\ 6\ 5$$

Shown practically for 13 ÷ 4, the answer is 3 remainder 1, or put another way, there are three whole groups and a remainder of 1. This remainder is one part towards a full group of 4, so is $\frac{1}{4}$. To show the remainder as a fraction, it becomes the numerator where the denominator is the divisor (the number that you are dividing by in the calculation).

3574 ÷ 8

$$8\overline{)35^{3}7^{5}4} \quad 4\ 4\ 6\ \text{r}6$$

6 ← remainder

8 divisor

So 3574 ÷ 8 is $446\frac{6}{8}$

Stage 7

To divide by a two digit number, long division will be used e.g.

6367 ÷ 28

$$28\overline{)6^{6}3^{7}6^{20}7} \quad 2\ 2\ 7\text{r}11$$

Key Facts box x 28

1x	28
2x	56
5x	140
10x	280

For 5x, it is easiest to first find 10x then halve the answer

Children may need to use jotting to help solve the calculation, these may not cover every step, so in the example above children may use jottings for the final step of 207÷ 28 for example:

140 (5x)	207
+56 (2x)	-196
196	11

Children should not be made to go onto the next stage if:

1) **they are not ready.**
2) **they are not confident.**

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.